METACOGNITIVE DECISIONS AND THEIR INFLUENCE ON PROBLEM SOLVING OUTCOMES

MERRILYN GOOS
The University of Queensland

In recent years, the role of metacognition in mathematical problem solving has begun to attract research interest, as metacognitive processes are considered to be an important factor influencing problem solving performance. The study described in this paper investigated the metacognitive strategies used by a pair of upper secondary school students while working on mechanics problems. The bulk of the data consisted of verbal protocols from uninterrupted think aloud paired problem solving sessions. Metacognitive decision points were identified in order to examine the monitoring contributions of each individual student, and the significance of student-student interactions. The main findings of the study were:
1. The subjects assumed differing, but complementary, metacognitive roles during problem solving.
2. The quality of the subjects' metacognitive decisions had an important influence on problem solving outcomes, but their decision making was sometimes adversely affected by the social interaction between them.

This paper reports on a study whose origins can be traced to the mathematics classroom, where the experience of teaching and observing upper secondary school students prompted questions such as:

> Why do students fail to use the knowledge they undoubtedly possess to help them solve mathematics problems that should be well within their grasp?

and

> Having chosen a problem solving strategy, why do students persist with the strategy whether or not it leads towards the desired goal?

Behaviours such as these can be explained by reference to the concept of metacognition, or awareness and regulation of one's own thinking (Brown, Bransford, Ferrara and Campione, 1983). Although it is important to be aware of one's mental state during problem solving, self-regulatory skills are considered to be even more crucial because the knowledge one possesses needs to be put to effective use. Regulation of cognition involves such activities as planning an overall course of action, selecting specific strategies, monitoring progress, assessing results, and revising plans and strategies if necessary (Garofalo and Lester, 1985).

Schoenfeld (1985a) identifies two broad types of control decision which may occur during the course of such activities, and which can influence problem solving outcomes. He argues that success is favoured if students:

1. exploit their knowledge to act on potentially useful information, and
2. discontinue inappropriate and unproductive strategies.

On the other hand, failure is virtually guaranteed by poor decisions like those made by the students mentioned above.

Early research on metacognition mostly involved reading or memory tasks (Brown et al., 1983), and it is only recently that the role of metacognition in the performance of mathematical tasks has begun to be studied. Much of this research has used either tertiary level or primary school students as the subjects (for example, Kroll, 1988; Venezky and Bregar, 1988); and studies which have attempted to train metacognitive strategies have tended to do so within separate "problem solving" courses (for example, Lester and others, 1989; Schoenfeld, 1985a), rather than treat metacognition - and problem solving itself - as a thinking process common to all branches of mathematics.

The present study differs from such research in two ways: the two subjects were senior secondary school students; and the problem tasks on which they worked, although challenging and unfamiliar, were similar to those they were likely to meet every day in their mathematics classroom. The aim of the study was to describe and qualitatively analyse the metacognitive strategies the two students used. Most of the data were obtained by instructing the students to think aloud while they worked cooperatively on these problems.

Two research questions were addressed:

1. What metacognitive strategies does each student use during problem solving?

The first question investigates the extent to which the subjects use the knowledge they possess to help them solve the problems they are set, and the manner in which they monitor and assess their progress towards the desired goal.

2. How does the presence, or absence, of metacognitive behaviour influence the outcome of problem solving?

The second question investigates Schoenfeld's (1983, 1985a) claim that the quality of metacognitive decision making can either promote or hinder problem solving success.

The remainder of the paper describes the conduct and results of the study, and discusses one important implication for mathematics teaching. However, as doubts are sometimes raised about the validity of verbal reports of thinking, and because think aloud methods have mainly been used with adults, rather than secondary school students like those who were the subjects of this research, it is appropriate to first outline the methodological decisions which influenced the design of the study.

VERBAL METHODS IN RESEARCH ON THINKING

Although the validity of verbal methods has been challenged on the grounds that subjects are unable to give accurate explanations for their behaviour (Nisbett and Wilson, 1977), verbal protocols do provide useful information if they are treated as data from which explanations should be inferred by the researcher, rather than the subjects (Genest and Turk, 1981).

The choice of suitable data collection procedures for this study was made by constructing three dimensions which can be used to characterise any verbal method. The dimensions are:

1. the time at which verbalisation is requested - concurrent or retrospective;
2. the degree of researcher intervention - from no intervention (in "think aloud" methods) to specific probes (in clinical interviews);
3. the instructions given to the subjects - to either report, or explain, their thinking (Ericcson and Simon, 1980; Genest and Turk, 1981).

After the advantages and limitations associated with variations along these dimensions were considered, it was concluded that the most accurate description of cognitive processes during task performance is obtained with concurrent verbalisation, no researcher intervention, and the instruction to report thinking. Nevertheless, two significant limitations remain: subjects may be unable to report all the cognitive processes of interest (incompleteness), and stress or task demands can distort cognitive processing (reactivity). For this reason it was decided to use Schoenfeld's (1985a, 1985b) pair protocol method, which has been designed to address these two limitations. Pair protocols are more likely to capture a complete record of students' typical thinking than single protocols because, first, two students working together produce more verbalisation than one and, second, the reassurance of mutual ignorance alleviates some of the pressure of working under observation. However, as incompleteness and reactivity are difficult to eliminate entirely, it is necessary to confirm inferences drawn from pair protocols by data from other sources, such as interviews and classroom observation.

METHOD

Subjects

The two subjects, "Rick" and "David", were Year 11 mathematics students in a large state run Senior College. The boys were sixteen years old when they took part in the study. Although both were described by their teacher as high ability students, David usually achieved better results in mathematics than Rick. This difference in status contributed an adversary flavour to their relationship; yet, the two remained good friends and consistently worked as a pair in the classroom. They were chosen for this study because they were highly articulate and accustomed to verbalising their thoughts as they worked together on mathematics problems.

Problems

To ensure that the problem tasks were relevant to the subjects' classroom experience, mechanics problems dealing with topics recently covered in their mathematics class were chosen for use in the think aloud sessions. For the purposes of this study, a task is considered to be a genuine problem for the student if progress is blocked at some stage, but merely an exercise for the solver who can call on a ready made solution schema (Silver, 1982). Most of the think aloud problems were therefore intended to be challenging enough to require, and elicit, metacognitive behaviour to remove any blockages. However, some routine exercises were also included to help put the subjects at ease at the start of each session.

Procedures

Three problem solving sessions, each of which lasted about one hour, were videotaped over a period of four weeks. The subjects attempted two problems in each session. Taping took place during the subjects' free time, in their regular mathematics classroom (which was otherwise vacant). The subjects were instructed to work together on the problems to produce one solution, and to say everything that came into their heads as they worked. They were informed that they would not be interrupted or given hints.

Retrospective interviews were used for supplementary data gathering if the pair protocols were found to be an incomplete record of the subjects' thinking. The interviews asked for a report, and then an explanation, of specific thinking processes so that the tentative inferences drawn from the pair protocols could be checked. Classroom observation (one and a half hours per week for ten weeks) and discussions with the subjects' teacher also allowed judgments to be made as to the typicality of the subjects' think aloud problem solving behaviour.

Data Coding and Analysis

Schoenfeld's (1985a) protocol analysis method was the basic tool for identifying the metacognitive strategies used by each student. The videotapes of each think aloud session were transcribed and the resulting protocols parsed into episodes representing distinctive types of problem solving behaviour: reading, analysis, exploration, planning, implementation and verification. However, a discussion of the episode parsing is beyond the scope of this paper, in which most interest centres on metacognitive decision points where new information was recognised or local assessments of specific aspects of the solution were made. Decision points were identified and classified in a way which extends Schoenfeld's analysis procedure. Unlike Schoenfeld's original scheme - which was designed to allow generalisations about the metacognitive behaviour of many pairs of students to be made - the analysis technique described below reveals the unique contributions made by two specific individuals, and the pattern of interactions between them.

Table 1. Scheme for Classifying Problem Solving Outcomes

Control Decision		Classification	Outcome
Discontinue inappropriate strategy	Exploit knowledge		
-	-	Control -ve	Bad decisions guarantee failure
+	-	Control neutral	Equivocal
+	+	Control +ve	Control decisions promote success
N/A	N/A	Expert	No need for control behaviour - the task is an exercise

Schoenfeld's new information points were renamed New Idea points (NI's), to describe places where previously overlooked or unrecognised information came to light or the possibility of taking a new approach was mentioned. NI's were classified according to who initiated them, how relevant or useful they were, and the nature and appropriateness of the response. Since any student's NI's may be either "useful" or "not useful", and his partner's response either "acceptance" or "rejection", it is reasonable to argue that if a good idea is rejected, or a bad idea accepted, then problem solving goes astray. In this way it was possible to trace studentstudent interactions, and their consequences for the solution process.

Local Assessments (LA's) of a particular aspect of the solution were classified according to who made them and what was assessed (procedure, result, task difficulty or knowledge), and were evaluated for appropriateness, effectiveness and influence on the solution.

The extent to which the presence, or absence, of metacognitive behaviour influenced problem solving outcomes was determined by classifying each protocol in terms of the control decisions it contained. The classification scheme summarised in Table 1 describes four types of solution attempt, and was derived from Schoenfeld's (1985a) discussion of the influence of control decisions on problem solving outcomes. Evidence for the two types of control decision, "discontinue inappropriate strategy" and "exploit knowledge", was provided by the incidence, function and quality of Local Assessments and New Idea points respectively.

RESULTS

This section summarises the results of analysis of four of the most interesting protocols - the PULLEY, GOLF, CRICKET and MASCOT problems. (Problem statements are given in the Appendix.) The findings are outlined by addressing the two research questions which guided this study. It should be noted, however, that these findings are results of a small case study and are not presented as conclusions which are generalisable from the sample to a larger population.

1. What metacognitive strategies does each student use during problem solving?

Table 2 shows the extent to which Rick (R) and David (D) exploited their knowledge and the manner in which they monitored their progress, by giving the numbers and types of New Idea points and Local Assessments initiated by each student across the four protocols.

Table 2. Summary of NI's and LA's - All Protocols

	PULLEY		GOLF		CRICKET		MASCOT	
Initiator	**R**	**D**	**R**	**D**	**R**	**D**	**R**	**D**
Total NI's	2	0	3	2	8	4	9	6
LA's								
procedure - accuracy	-	-	-	-	1	-	3	-
procedure - usefulness	-	1	-	-	1	4	4	2
result - accuracy	-	-	-	1	2	2	1	-
result - reasonableness	-	-	-	-	1	-	1	2
task difficulty	-	2	-	1	-	1	-	1
knowledge	-	-	-	1	-	1	-	3
Total LA's	0	3	0	3	5	8	9	8

Several inferences can be made about their metacognitive strategy use:

1. Rick consistently generated more new ideas than David.
2. David produced more Local Assessments than Rick (except in the MASCOT protocol).
3. Only Rick checked the accuracy of procedures as they were executed.
4. Only David evaluated task difficulty, and assessed what was known or not known.
5. Rick and David shared the responsibility for assessing the accuracy and reasonableness of results.

It seems, then, that Rick and David have differing, but complementary, metacognitive strengths. Rick played two roles during problem solving: he was both the idea generator and the checker of David's calculations. However, many of Rick's ideas were irrelevant or unworkable. Because he failed to assess the usefulness of his ideas, Rick was in constant danger of setting off on wild goose chases. The task of rescuing him from this fate fell to David, who effectively filled the role of procedural assessor in all but the MASCOT problem. (In the latter protocol it was Rick who made the majority of procedural assessments as he tried, unsuccessfully, to convince David that his strategy was wrong.)

2. How does the presence, or absence, of metacognitive behaviour influence the outcome of problem solving? Table 3 classifies each protocol according to the influence of control decisions on the outcome.

Table 3. Influence of Metacognitive Behaviour on Problem Solving Outcomes

PROBLEM	OUTCOME	DISCONTINUE STRATEGY	EXPLOIT KNOWLEDGE	CLASSIFICATION
PULLEY	Success	N/A	N/A	Expert
GOLF	Success	+	–	Control Neutral
CRICKET	Success	+	+	Control +ve
MASCOT	Failure	–	–	Control –ve

The PULLEY problem, presented as a warm up task at the start of the second videotaped problem solving session, was solved in a little over five minutes. It elicited very little metacognitive behaviour because the students were able to call on a ready made solution schema. Since control decisions had no effect on the outcome this task was merely an exercise for Rick and David, and the protocol is classified as expert.

Although the GOLF problem was also intended to be an exercise, Rick and David's progress was blocked for a time because their inappropriate strategy had led to an impossible result: 2.5 = cos ∴. Despite their previous experience with trigonometry, the students continued until their calculator's "Error" message alerted them to their mistake. After mechanical errors were eliminated as the cause of the difficulty, more careful reading and analysis pinpointed the source of the error and the incorrect strategy was discontinued. Nevertheless, the pair's failure to access their knowledge of trigonometry could have had serious consequences, and the GOLF protocol is therefore classified as control neutral: the outcome could have been either success or failure.

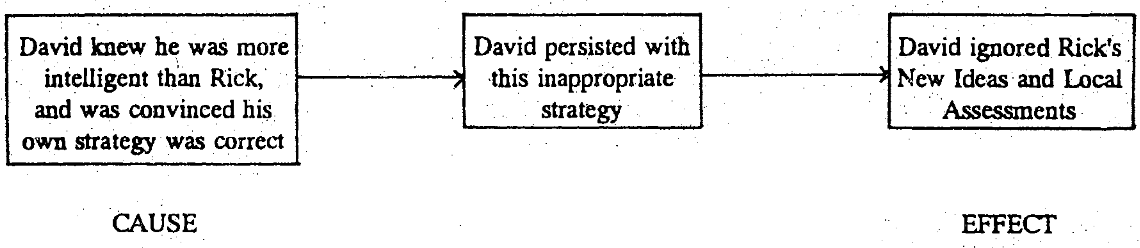
In the CRICKET problem, Rick and David effectively coordinated their respective roles of idea generator/calculation checker and procedural assessor. On discovering that they were stuck after only two minutes of working, David prevented Rick from going any further with his inappropriate strategy for calculating the time of flight of the ball. During the next few minutes, many new ideas were proposed, tested, and rejected as unhelpful. Relevant information was eventually recognised, and prior knowledge about projectile motion was accessed and used. Good control decisions promoted success; therefore the CRICKET protocol is classified as control positive.

The MASCOT problem was the only one that Rick and David were unable to solve. After analysing the problem statement they settled on an inappropriate and unproductive trigonometry based strategy, which they nevertheless pursued for more than twenty minutes. Evaluation of student-student interactions revealed many instances where good ideas were rejected or ignored, and bad ideas were accepted without assessment. Useful knowledge about the relationship between force and acceleration remained unexploited, and the wild goose chase was not curtailed. Because these poor control decisions contributed to Rick and David's failure to solve the MASCOT problem, the protocol is classified as control negative.

DISCUSSION AND IMPLICATIONS

The first finding of the present study, concerning metacognitive strategy use by secondary school students, adds to the limited knowledge that currently exists in this area. By coordinating their differing strategic preferences and metacognitive strengths, Rick and David were usually able to think their way around obstacles to their progress. This result illustrates the benefits of peer collaboration as an effective means of developing and practising selfregulation in the problem solving classroom (Schoenfeld, 1987).

However, further consideration of data related to the second finding suggests that caution is needed in forming collaborative groups. Although the quality of metacognitive decision making did indeed contribute to problem solving success or failure, just as Schoenfeld (1983, 1985b) claims it should, there is evidence that the outcome of the MASCOT problem was influenced by another factor - the social interaction between the subjects. In particular, Rick's useful New Ideas and appropriate Local Assessments were consistently rejected or ignored by his partner. A plausible explanation for Rick and David's failure to solve this problem could therefore involve the following chain of cause-effect relationships:



In the MASCOT problem, the supposedly collaborative interactions between the students hindered, rather than promoted, metacognitive decision making.

Clearly, not all collaborative relationships are educationally valuable. Forman (1989) names three conditions for collaboration to be effective:

1. Students must have mutual respect for each other's perspective on the task.
2. There must be an equal distribution of knowledge.
3. There must be an equal distribution of power.

All three conditions were violated in the MASCOT protocol. First, David did not respect Rick's New Ideas or Local Assessments. Second, there were unequal distributions of knowledge and power: David was the more powerful student because he took charge of the course of the solution; but he was not, in this protocol, the more knowledgeable. David rejected Rick's attempt to usurp his own role of procedural assessor (see Table 2), and the continued existence of differentiated problem solving roles had an adverse effect on the solution process. Further research is needed on the conditions under which peer interaction fosters or interferes with metacognitive self-regulation, so that effective approaches for teaching problem solving in small group settings can be developed.

REFERENCES

Brown, A. L., Bransford, J. D., Ferrara, R. A. & Campione, J. C. (1983). Learning, remembering and understanding. In P. H. Mason (Ed.), **Handbook of Child Psychology, Vol. 3** (4th ed.) (pp. 77-166). New York: Wiley.

Ericcson, K. A. & Simon, H. A. (1980). Verbal reports as data. **Psychological Review, 87,** 215-251.

Forman, E. (1989). The role of peer interaction in the social construction of mathematical knowledge. **International Journal of Educational Research, 13,** 55-70.

Garofalo, J. & Lester, F. K., Jr. (1985). Metacognition, cognitive monitoring, and mathematical performance. **Journal for Research in Mathematics Education, 16,** 163-176.

Genest, M. & Turk, D. (1981). Think-aloud approaches to cognitive assessment. In T. V. Merluzzi, C. R. Glass & M. Genest (Eds.), **Cognitive Assessment** (pp. 233-269). New York: The Guilford Press.

Kroll, D. L. (1988). Cooperative problem solving and metacognition: A case study of three pairs of women. Doctoral dissertation, Indiana University. Dissertation Abstracts International, 49, 2958 A. (University Microfilms No. 8902580).

Lester, F. K., Jr. and others (1989). The role of metacognition in mathematical problem solving: A study of two grade seven classes. Final Report. Indiana University, Bloomington: Mathematics Education Development Centre. (ED 314 255).

Nisbett, R. E. & Wilson, T. D. (1977). Telling more than we can know: Verbal reports on mental processes. **Psychological Review,** 84, 231-259.

Schoenfeld, A. H. (1983). Beyond the purely cognitive: Belief systems, social cognitions, and metacognitions as driving forces in intellectual performance. **Cognitive Science, 7,** 329363.

Schoenfeld, A. H. (1985a). **Mathematical Problem Solving.** Orlando, Florida: Academic Press. Schoenfeld, A. H. (1985b). Making sense out of "out loud" problem-solving protocols. **Journal of Mathematical Behaviour, 4,** 171-191.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), **Cognitive Science and Mathematics Education** (pp. 189-215). Hillsdale, New Jersey: Erlbaum.

Silver, E. A. (1982). Knowledge organisation and mathematical problem solving. In F. K. Lester, Jr. & J.

Garofalo (Eds.), **Mathematical Problem Solving: Issues in Research** (pp. 15-25). Philadelphia: The Franklin Institute Press.

Venezky, R. L. & Bregar, W. S. (1988). Different levels of ability in solving mathematical word problems. **Journal of Mathematical Behaviour, 7,** 111-134.

APPENDIX

PULLEY

Two bodies of mass 4 kg and 3 kg are at rest on two smooth inclined planes placed back to back. The bodies are connected by a string passing over a smooth pulley at the top of the planes. If the 4 kg mass rests on a plane inclined at 35° to the horizontal, find the inclination of the other plane.

GOLF

A golfer hits a ball from a point on a level fairway, and 2 seconds later it hits the fairway 50 meters away. Find:

(a) the velocity and angle of projection of the golf ball

(b) the maximum height of the ball above the fairway.

CRICKET

A batsman hits a cricket ball "off his toes" towards a fieldsman who is 65 meters away. The ball reaches a maximum height of 4.9 meters and the horizontal component of its velocity is 28 m/s. Find the constant speed with which the fieldsman must run forward, starting at the instant the ball is hit, in order to catch the ball at a height of 1.3 meters above the ground. (Use $g = 9.8$.)

MASCOT

A mascot suspended from a car's rear view mirror hangs vertically when the car is moving with uniform velocity of 80 km/hr along a straight level road. The brakes are applied so that the car is stopped with uniform retardation. Find the angle through which the mascot is deflected if the car comes to rest 137 meters after the brakes are applied.